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DECENTRALIZABILITY OF TRUTHFUL REVELATION AND OTHER DECISION RULES*

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ABSTRACT

The revelation principle in welfare economics asserts that, in principle, the allocative performance of <u>direct mechanisms</u> (institutions in which an exhaustive report of privately held information is elicited from each agent) is at least as good as that of any other class of institutions. Recently it has been suggested that direct mechanisms may be of practical as well as theoretical importance, although in practice agents would supply only summary information. The distinction between the incentive properties of exhaustive and summary reporting is studied here. The problem of inducing a single person truthfully to report the value of a continuous multidimensional parameter is examined. It is shown that, unless strong restrictions are placed on the person's utility function, the incentive properties of truthful summary reporting about the parameter (e.g., reporting of its first coordinate) are extremely fragile. This assertion of fragility is given a precise statement in terms of topological notions.

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1. INTRODUCTION

An outstanding accomplishment of welfare economics of the past several decades has been the development of a systematic approach to problems of incentives in environments where persons have private information. One of the central insights upon which this approach is based is that the theoretical analysis of the economic role of information can be accomplished without having to examine case by case the bewildering variety of actual or possible economic institutions. Rather, results established with reference to a special class of institutions called *direct mechanisms* are in fact completely general. It is not necessarily supposed that any direct mechanisms have ever been observed in actual economic life; only the theoretical assertion is made that institutions of this form exist which in principle would duplicate the performance of actual ones. Direct mechanisms work by eliciting from each person an exhaustive report concerning his private information, and by defining an allocation on the basis of these reports. The private information held by the various persons, taken together, is called the *state*.

The assertion that direct mechanisms exist which provide allocations that are as desirable as those provided by other institutions is known as the *revelation principle*. According to this principle, if a state-contingent allocation can be achieved as the equilibrium outcome of any institution, then it can be achieved as the outcome of an institution in which (a) each participant is required to give a report of his private information, and (b) for all participants to give truthful and complete reports is an equilibrium.[1]

Recently it has been suggested that the revelation principle may serve as a practical basis for the design of particular institutions such as auctions or regulatory procedures. This suggestion marks a significant change from viewing direct mechanisms as a theoretical construct to viewing them as candidates for actual use. It is natural that direct mechanisms should be considered in situations where private information is not very complicated or where exhaustive reporting of private information is feasible and inexpensive, but the use of these mechanisms raises questions in situations where these conditions are not satisfied. For instance, it is sometimes charged that laws mandating disclosure by firms of financial information impose significant costs which are inefficient because the detailed accounting information which must be furnished is far in excess of what investors require. According to this argument, full reporting of firm managers' private information is undesirable even though it is feasible, because the cost of this reporting exceeds the benefit which is obtainable from it. The argument against mandatory financial disclosure continues by suggesting that negotiation between firm owners and managers will result in an efficient agreement about what information should be reported, and it concludes by appealing to the revelation principle to assert that this agreement would be implementable by an appropriately chosen system of incentives.

Regardless of whether the earlier stages of this argument are sound, its last step is faulty. While the critics of financial disclosure laws assume that the summary information needed by stockholders will be provided truthfully in its condensed form, the revelation principle actually does not guarantee that incentives exist which would induce truthful revelation of this summary information. The assumption must rely on some implicit premise that the same incentives which would induce truthful complete reporting could be modified to induce truthful summary reporting instead. The present paper deals precisely with the question of whether this implicit premise is warranted. The conclusion will be tentatively negative. It will be argued that, unless strong restrictions can be put *a priori* on the preferences of an agent subject to incentives, the provision of a schedule of incentives which induce truthful summary reporting is practically an impossible task. The following four remarks provide a more precise statement of this result and indicate how it is related to the revelation principle as usually presented.[2]

Remark 1: The revelation principle is formulated in terms of a game with several players. Each player makes a decision which will

be best for himself, given the structure of the game and his expectations about what the other players will do. The present paper concerns the problem of inducing a single person to give a truthful and complete report of his private information. This problem differs in two respects from the problem addressed by the revelation principle. First, because one person is considered in isolation, only an optimal decision rather than an equilibrium needs to be characterized. Second, while the revelation principle asserts that a game having specific payoffs in equilibrium (i.e., an allocation in every state which is as good for the players as that provided by the game initially considered) must have truthful reporting as an equilibrium, this paper asks instead whether there is any system of incentives at all which will induce truthful reporting. Despite these differences, it seems very likely that the present results will have analogues for many-person environments.

Remark 2: According to the revelation principle, a person is considered to have incentive to perform an action in some state of nature if the action is *among* his utility-maximizing actions in that state. Thus, if the person's action does not enter directly into his utility function (e.g., if it is to send a message rather than to perform some arduous task), simply to give a fixed reward regardless of what is done will provide incentives in this weak sense for any action. It might further be required that the action chosen in any state be the *unique* utility-maximizing action in that state. When it applies simultaneously to all the players in a game, this stronger requirement is commonly known as _decentralization_. Whether truthful revelation (and other _decision rules_ or state-contingent actions) can be decentralized is the topic of this paper. Because only the decision problem of a single person will be studied explicitly here, a schedule of rewards will be said to _strictly support_ a decision rule if it provides incentives relative to which the rule always specifies the decision maker's unique best action.

Remark 3: Implicit in the suggestion that the revelation principle might be of practical importance is the idea that a wide variety of incentive problems can be solved indirectly by being reduced to the canonical problem of inducing truthful revelation. However, sufficient conditions for the canonical problem to have a solution (i.e., for truthful revelation to be decentralizable) have been given only when private information is represented by a state space which is either a finite set or a one-dimensional continuum. Intuitively, persons' private information in actual situations seems more complex than these representations would permit. In the present paper, a local version of strict supportability will be studied for decision rules defined on a multidimensional state space. It will be asked whether decision rules are locally strictly supportable for a class of utility functions comparably general to the class for which comparative-statics results hold in price theory. (Technically, are they strictly supportable for an open set of utility functions in the Whitney C^2 topology?)

Theorem 1 will establish that, in many environments, truthful

exhaustive revelation and related decision rules have this property. Theorem 2 will show further that, in a natural sense, the existence of reward schedules which locally strictly support truthful exhaustive revelation is a simple consequence of the ability of the reward giver to affect differentially the opportunity costs of the person receiving incentives for performing various actions (i.e., for reporting truthfully or for giving various untruthful reports), and that it does not depend on delicate considerations having to do with special features of the incentive taker's preferences regarding particular rewards or sanctions. These theorems show that the requirement of strict supportability (rather than of decentralizability as usually formulated) and the restriction of reward schedules to the class of smooth functions do not _per se_ rule out the possibility of deriving affirmative results about decentralization of decision rules.

The main result of this paper is Theorem 3. This theorem will state that, for almost all possible preferences of the incentive taker (i.e., for a residual set of utility functions in the Whitney C^2 topology), the local strict supportability of summary reporting is extremely sensitive to the kinds of special considerations which do not arise with respect to exhaustive reporting. Taken together, Theorems 2 and 3 show that the incentive properties of truthful summary reporting are immensely more fragile than are those of truthful exhaustive reporting. Interpreted in this way, Theorem 3 appears as a strongly negative result which casts severe doubt on the informal interpretation of the revelation principle to assert that truthful summary reporting of information can be decentralized.

The results of this paper are presented in a way which emphasizes the complete analogy between incentive problems which concern truthful reporting and those which involve other kinds of actions. Any smooth decision rule will be seen to be locally isomorphic on a dense open set of states either to truthful exhaustive reporting, or to truthful summary reporting, or to a rule specifying that some particular action should always be taken regardless of the state. In light of Theorems 2 and 3, characterization of the local incentive properties of constant decision rules suggests itself naturally as a problem. Theorem 4 addresses this issue. It will state that constant rules share the favorable attributes of truthful exhaustive reporting.

Remark 4: In this paper, the decision maker's preference over rewards is described directly rather than derived from some more fundamental preference relation. For instance, if the owners of the firm were to offer a compensation package involving both salary and stock options in order to provide managerial incentives, then the manager would be described in terms of a reduced-form utility function over combinations of money and stock. In general, such a function will not be derivable from maximization of expected utility of the stochastic wealth which money-plus-stock combinations will yield. Furthermore, even restrictions such as separability which can be placed directly on the utility function for rewards will not be considered here.

The assumption that the manager maximizes expected utility of wealth, or some other a priori restriction on his preferences over the rewards which may be provided, might reverse the conclusions which are derived here in the absence of restrictions. The present results will imply that reporting in full, but not summary reporting, can reasonably be expected to be induced by an incentive schedule if no special assumptions about utility are made. This might be regarded as a statement about economic modeling, rather than as a hard fact about the world. The point of this paper is to show that parametric assumptions which have been made in the literature on incentives, and which have been treated as being mere technical conveniences, perhaps crucially affect the conclusions that are drawn. The conclusions of this literature are not necessarily to be rejected, but the basis on which they rest deserves closer scrutiny than it has so far received.

2. AN EXAMPLE

Before proceeding to the technical results of the paper, the intuition which lies behind them will be discussed in terms of an example pertaining to the issue of financial disclosure which has already been mention in the Introduction. Specifically, we refer to the work of Ross [10] on signalling equilibria in stock markets. Ross poses the question, is there some action which firm managers can be induced to take which will systematically reveal their inside information to shareholders? He answers this question affirmatively in the context of a partial equilibrium model (i.e., one in which the relation between return distributions of different firms is ignored), under the assumption that the distribution of firms' returns belongs to a monotone-likelihood family. The assumptions made in this model preclude consideration of the relationships between risk and return which are of central concern in financial theory. It is fair to characterize them as being highly restrictive.[3]

In truth, the market valuation of financial assets seems to be quite complicated. Let us consider specifically the pricing of equity. One might presume that the annual reports which firm managers send to their stockholders contain sufficient information so that investors can determine the value of equity at the time the reports are released, if that information is provided truthfully. (At least, this seems to be what is presumed by some currently popular arguments for the redundancy of financial disclosure laws.) However, the annual reports contain only a miniscule part of managers' total information about the current operations, plans, and forecasts of their firms. Thus the single question which can be posed about managerial incentives in the context of Ross's simplified model splits into two very distinc questions with respect to actual markets. First, it may be asked whether the manager of a firm can be induced to give a truthful report _in full detail_ of what he knows. Truthful revelation in this sense is the goal of the exhaustive reports (such as the 10K report in the United States) mandated by disclosure laws. Alternatively, it may be asked whether the manager can be induced to give a truthful _summary_ report of his private information. That is,

is it possible to formulate incentives which will lead managers to provide accurate information in their annual reports to stockholders, without requiring them to provide any further information in addition to this? It is conceivable that incentives for truthful exhaustive reporting might be available, while incentives for summary reporting were not. In this case arguments against the efficiency of disclosure laws would need to be based on some sort of detailed cost-benefit calculations, rather than on any simple premise about sufficiency and accuracy of information provided under _laissez-fare_.

It will be shown in the succeeding sections of this paper that to substitute summary reporting for complete reporting will dramatically increase the complexity and delicacy of the incentive problem for a large class of agents' preferences. This result may initially seem paradoxical, since it is natural to view complete reporting as asking the agent to do more than summary reporting does. However, reflection on the example of managerial reporting will illustrate the phenomenon that additional detail in reporting provides opportunity for increased flexibility in incentive design. Suppose that the annual report to stockholders contains an accounting statement of current profits and also a projection of next year's profits. If investors are certain that these two summary numbers acurately reflect the information of the manager, they may not care how current revenue has been allocated between operating expenses, depreciation and profit. Nevertheless, even though they do not need to use this breakdown in order to make their own portfolio decisions, owners might find it valuable because it could enable them to deter the manager from overstating profits.[4] Even if they cannot verify the disaggregated information directly, they may yet find it a useful basis on which to condition the manager's compensation. This insight explains the marked contrast between the results now to be stated.[5]

3. STATEMENT OF THE MAIN RESULTS

We first describe the formal theory in which results are to be formulated: Let S be a set of <u>states</u>, A be a set of <u>actions</u>, and T be a set of <u>transfers</u>. These sets will be considered as open subsets of Euclidean spaces of dimensions k, m and n, respectively. (In greater generality, they could be smooth manifolds (cf. [1,Def.I.1.6]) of those dimensions.) $\mathbb{R}$ denotes the real numbers.

The state represents the private information of a person with utility function $u:S \times T \to \mathbb{R}$. Note that the person's action does not enter <u>per se</u> into his utility function. U will denote the set of utility functions which are smooth (i.e., infinitely differentiable) everywhere on $S \times T$. If $n > 1$, then a transfer will be a consumption bundle which may include several different commodities. Alternatively u might be interpreted as an indirect utility function, and a transfer might be taken to be a budget (i.e., a price-income pair) made available to the person.

A <u>decision rule</u> is a smooth mapping $d:S \to A$. The interpretation of the rule d is that the person would choose action d(s) in each state s. D will denote the set of decision rules.

A <u>reward schedule</u> is a smooth mapping $r:A \to T$. The interpretation of the schedule r is that the person will receive transfer r(a) (from some donor not explicitly represented in the model) if he chooses action a. R will denote the set of reward schedules.

Reward schedule r <u>strictly supports</u> decision rule d for utility function u at state $\bar{s}$ if, for all states s in some neighborhood V of $\bar{s}$, d(s) is the unique action a in A which maximizes u(s,r(a)). This condition is stronger than (the local version of) the usual criterion of when a reward shcedule implements a decision rule, because of the uniqueness requirement. This requirement is made because (since the person's action is not an argument of his utility function) the trivial reward schedule which assigns a constant transfer regardless of the person's action would otherwise implement every decision rule for every preference. In the applied literature on incentive problems, reward schedules which strictly support decision rules (rather than which merely implement them in the weaker sense) are virtually always sought.

Decision rule d is <u>strictly supportable</u> for utility function u at state $\bar{s}$ if some reward schedule in R strictly supports d for u at $\bar{s}$. Strict supportability suggests that the giver of incentives must solve a local problem, but that the decision-maker whome the incentives affect is solving a global problem of optimization. The local problem of the incentive-giver has an intuitive interpretation: It is that, if the incentive-giver had sufficiently precise information that the true state was close to $\bar{s}$, then he would be able to strictly support the decision rule relative to the set of states not excluded by this information.

Truthful reporting is now defined. It is assumed that there is a smooth embedding $d_*: S \to A$ in D. (cf. [2,Def. I.2.3] Note that this assumption implies $k \leq m$.) The action $d_*(s)$ will be regarded as a report of state s, so that d_* defines truthful reporting as a decision rule.

The feature of d_* which will be crucial for the results of this paper is that it is an <u>immersion</u> at each state. (I.e., its Jacobian has rank k at each state.) For any decision rule d, let I(d) be the set of states at which d is an immersion. Theorem 1 states that, at a state where it is an immersion and under the assumption that the space of transfers has sufficiently high dimension,a decision rule is locally strictly supportable for utility functions which are strictly concave in transfers, which have a most preferred transfer at the state in question, and which exhibit the greatest possible interaction between preferences among states and among transfers.

<u>Theorem 1</u>: If $\bar{s} \in I(d)$ and either $k = m \leq n$ or else both $k < m$ and $k < n$, and if utility function u satisfies

At every s and t, $u_{TT}(s,t)$ is negative definite, (1)

For some $\bar{t} \in T$, $u(\bar{s},\bar{t}) = \max_{t \in T} u(\bar{s},t)$, and (2)

$u_{TS}(\bar{s},\bar{t})$ has rank k. (3)

then there is a decision rule with $r(\bar{s}) = \bar{t}$ which strictly supports d for u at $\bar{s}$. In particular, under either dimensionality assumption, the theorem applies to truthful and complete reporting (decision rule d_*).

When S and A have dimension one, and T is two-dimensional (e.g., transfers are combinations of a premium level and a level of self insurance, or of a wage and a required level of schooling, etc.), then Theorem 1 bears close relation to a standard technique for constructing reward schedules which strictly support various decision rules arising in applied incentive problems: For any $\bar{t}$ at which $u_T(\bar{s},\bar{t})$ is nonzero, define X to be the intersection of T with the line perpendicular to this vector (or more generally, to be a one-dimensional smooth manifold containing $\bar{t}$, at which point the tangent is perpendiular to $u_T(\bar{s},\bar{t})$, e.g., the zero-profit locus for a competitive incentive giver). Then the assumptions of Theorem 1 are satisfied with respect to X as the space of transfers, so a decision rule d with nonzero derivative at $\bar{s}$ is locally strictly supportable at $\bar{s}$. The proof of Theorem 1 will generalize the familiar method of finding a reward schedule r which accomplishes this by expressing r as the solution of a differential equation with initial condition $\bar{t} = r(d(\bar{s}))$. What is most noteworthy about this method is that, in practice, there is no restriction on which transfers may be chosen to define the initial condition. The economic intuition which explains this phenomenon is that the sufficient condition for local strict supportability is stated in terms of opportunity costs of performing alternative actions, and that varying the initial condition merely changes the base from which opportunity costs are calculated. Theorem 2 asserts that this intuition is good in higher dimensions as well, as long as the decision rule is an immersion.

Formally, if $T(d,\bar{s},u)$ denotes the set of transfers t such that there exists a reward schedule r which locally strictly supports d for u at $\bar{s}$ and which satisfies $r(d(\bar{s})) = t$, then it is typical to find that $T(d_*,\bar{s},u) = T$ for incentive problem obtained when the applied problem in question is reformulated in terms of the revelation principle. Formally, what it will mean for a phenomenon to be typical is that it should occur with respect to all utility functions in some subset of U having nonempty interior in the Whitney C^2 topology.[6] This is actually a rather fine topology, so Theorem 2 is not particularly strong, but the fineness of this topology will make more forceful the negative result to be stated in Theorem 3.

<u>Theorem 2</u>: If $\bar{s} \in I(d)$ and either $k = m \lesssim n-1$ or else both $k < m$ and $k < n-1$, and if the utility function u satisfies

$$\text{At every } s \text{ and } t,\ u_{TT}(s,t) \text{ is negative definite, and} \tag{4}$$

$$\text{At every } t, \text{ the matrix } [u_T(\bar{s},t), u_{TS}(\bar{s},t)] \text{ has rank } k+1. \tag{5}$$

then $T(d,\bar{s},u) = T$. The set of all utility functions satisfying (4) and (5) has nonempty interior in the Whitney C^2 topology on U.

A less restrictive condition than being an immersion at a point is to be a <u>local submersion</u>. The smooth decision rule $d:S \to A$ is a local submersion at s if there i so e neighborhood V of s such

that the rank of the Jacobian matrix of d at s is the maximum of its rank at any point in V, and is at least one.[7] L(d) will denote the subset of S consisting of points where d is a local submersion. If $s \in L(d)$, then the implicit function theorem may be applied to d at s. $I(d) \subseteq L(d)$ by [2, Prop.I.2.10]. Theorem 3 asserts that the incentive properties of a decision rule d at states in $L(d)\setminus I(d)$ are enormously more fragile than at states in I(d). In particular, the property guaranteed on I(d) by Theorem 2 is asserted by Theorem 3 to fail in a spectacular way.

<u>Theorem 3</u>: Let $\bar{s} \in L(d)\setminus I(d)$. Then the set of utility functions u, for which the complement of $T(d,\bar{s},u)$ contains a residual subset of T, contains a subset which is residual in U in the Whitney C^2 topology. Since both T and U possess the Baire property, the set of utility functions u for which $T(d,\bar{s},u)$ contains a nonempty open subset of T has empty interior in U.

<u>Remark 5</u>: Suppose that $j < k$, and consider <u>summary reporting</u> of the first j coordinates of s (i.e., $d(s) = (s_1,\ldots,s_j,0,\ldots,0)'$) as a decision rule. Then $L(d) = S$, so Theorem 3 applies to summary reporting at every state.

<u>Remark 6</u>: Theorem 3 extends easily to situations in which the action taken is an argument of the utility function. (I.e., $u:S \times A \times T \to \mathbb{R}$.) In this case, define $T_A = A \times T$ and define $R_A = \{r:A \to T_A \mid a \quad t\ r(a) = (a,t)\}$. That is, R_A is the set of reward functions which dictate that the person will experience the direct consequences of the action taken as well as receiving a transfer.

Since R_A is a subset of the set of all smooth functions from A to T_A, and since the domain of u may be regarded as S X T_A, Theorem 3 holds in this setting.

Once again, consider the contrast between the economic implications of Theorems 2 and 3. Assume that firms' profit distributions belong to a two-parameter family. Consider a firm which is confidently (but not with absolute certainty) believed <u>ex ante</u> by its shareholders to earn an expected profit P distributed with variance V, and suppose that the shareholders wish to pay a market-determined compensation C to the manager if P and V do in fact characterize the statistical distribution of profits and that they also desire to induce the manager truthfully to report the profit distribution. If the manager's preferences satisfy (4) and (5), then by Theorem 2 the shareholders' two objectives can be simultaneously attained. (At least, in the sense of local strict support. Formally, S is the set of possible P-V combinations, A is a copy of S and the decision rule is the identity mapping, and T might (in order to have dimension 2 as required for the theorem to apply) be a set of randomized compensations — for instance, combinations of a certain salary and a stock option, the value of which will be affected by random market fluctuations.) Now suppose that the manager receives a bequest B, which would be represented as a constant function from A to T which is added to whatever reward schedule the shareholders offer. (I.e., if he gives report (P',V'), the manager receives r(P',V') + B.) Except under the restrictive assumption that the manager has constant

absolute risk aversion, the reward schedule initially offered by the shareholders would presumably no longer elicit truthful revelation. However, again by Theorem 2, the shareholders could offer a new reward schedule which would still provide compensation C (and hence gross reward C + B) for giving report (P,V) and which would locally strictly support truthful reporting in the presence of the bequest.

In contrast, consider the situation which is identical except that the shareholders desire only a summary report of the statistically expected profit rather than an exhaustive report of the profit distribution. Now, by Theorem 3, for most utility functions satisfying (4) and (5) (i.e., on the intersection of the set of such functions with a residual set) there are arbitrarily small bequests (i.e., combinations of a sum of money and a stock portfolio) which will make it impossible for the shareholders to pay the market-determined compensation in the event that their prior assessment of the profit distribution is correct and yet locally strictly to support truthful summary revelation by the manager of the expected profit level. Because of its extreme sensitivity to these wealth effects, the local strict support of truthful summary reporting would seem to be practically unattainable.

It is a consequence of Lemma 2 (to be stated below) that, for any decision rule d, L(d) and the set of states at which d is locally constant jointly comprise a dense open subset of S. For the sake of completeness, the incentive characteristics of d on the latter set are now described.

<u>Theorem 4</u>: If d is constant on a neighborhood of $\bar{s}$, then $T(d,\bar{s},u) = T$ for all utility functions u in a Whitney C^2-open subset of U.

4. PROOFS OF THEOREMS 1, 2 AND 4

To begin, some notation regarding vectors and derivatives of functions will be defined. $\underline{0}$ will denote the origin of $\mathbb{R}^h$. Vectors will be columns, and z' will denote the transpose of z. Superscripts will denote coordinate projections of a function with multidimensional range. (E.g., if $f:Z \to \mathbb{R}^h$, then $f = (f^1,\ldots,f^h)'$ and $f^i:Z \to \mathbb{R}$ for $i \leq h$.) Subscripts will denote partial derivatives. (E.g., if $Z \subseteq \mathbb{R}^j$ and $i \leq j$, then $f_i(\bar{z}) = (\frac{\partial}{\partial z_i} f^1(\bar{z}),\ldots,\frac{\partial}{\partial z_i} f^h(\bar{z}))'$.) A capital letter denoting a space, used as a subscript, will denote the matrix of partial derivatives with respect to coordinates in that space. The derivative of a scalar function will be a column vector (e.g., $u_S(s,t) = (u_1(s,t),\ldots,u_k(s,t))'$), and the derivative of a vector function will be a matrix, the columns of which are derivatives with respect to coordinates (e.g., $u_{ST}(s,t) = [v_{k+1}(s,t),\ldots,v_{k+n}(s,t)]$, where $v(s,t) = u_S(s,t)$).

The implicit function theorem will be used in the following form:

<u>Lemma 1 [2, Theorem I.2.4]</u>: Let $W \subseteq \mathbb{R}^h$ and $Y \subseteq \mathbb{R}^j$ be open sets, and suppose that $f:W \times Y \to \mathbb{R}^j$ is smooth, $f(\bar{w},\bar{y}) = \bar{0}$, and $f_Y(\bar{w},\bar{y})$ is nonsingular. Then there is a neighborhood V of $\bar{w}$ and there is a smooth function $g:V \to Y$ such that $f(w,g(w)) = \bar{0}$ for every $w \in V$.

Furthermore, $g_W(w)$ has the same rank as $f_W(w,g(w))$.

Another lemma which will be used repeatedly is stated now as well.

<u>Lemma 2 [8, Thm 3.13.1, Thm 3.13.2]</u>: Let $W \subseteq \mathbb{R}^h$ and $Y \subseteq \mathbb{R}^j$ be open, and let $f:W \to Y$ be smooth. Then at each w, $\{v \mid \text{rank } f_W(v) \geq \text{rank } f_W(w)\}$ is a neighborhood of w. If there is some neighborhood V of w such that $\text{rank } f_W(w) = \max_{v \in V} \text{rank } f_W(v)$, then there is a submanifold Z of Y which contains f(w), which has dimension equal to rank $f_W(w)$, and such that f maps a neighborhood of w onto Z.

Now the proof of Theorem 1 is begun. The idea is first to define a reward schedule on a set of actions which the decision rule prescribes in states close to $\bar{s}$, and then to extend the domain of definition to all of A in a way which preserves the incentive properties of the subschedule.

<u>Lemma 3</u>: If u satisfies (1), (2) and (3), then there is a neighborhood V of $\bar{s}$ and an embedding $b:V \to T$ such that

$$\forall s \in V \quad u(s,b(s)) = \max_{t \in T} u(s,t) \tag{6}$$

<u>Proof</u>: In Lemma 1, take $W = S$, $Y = T$, and $f = u_T$. Lemma 1 applies because, by (1), f_T is nonsingular at every (s,t). By (2), $f(\bar{s},\bar{t}) = u_T(\bar{s},\bar{t}) = \underline{0}$). Define $b(\bar{s}) = \bar{t}$ and $b(s) = g(s)$, where g is the function asserted to exist by Lemma 1. By the concluding statement of Lemma 1, $b_S(s)$ has the same rank as $u_{TS}(s,b(s))$ at every

s in the domain of b. Thus, by (3) and Lemma 2, the restriction of b to some neighborhood of $\bar{s}$ is an immersion. By [2, Prop.I.2.10], there is a neighborhood V of $\bar{s}$ such that b is a diffeomorphism from V to b(V), which is a submanifold of T. I.e., the restriction of b to V is an embedding. For simplicity, V will be regarded as the domain of b.

It remains to show (6). By (1), u(s,t) is a strictly concave function of t for every s. Thus the first-order necessary condition for b(s) to maximize u(s,t) is in fact sufficient. This condition is precisely that $u_T(s,b(s)) = \underline{0}$, which is assured by construction of b. Q.E.D.

<u>Lemma 4</u>: If $\bar{s} \in I(d)$ and u satisfies (1), (2) and (3), then there is a k-dimensional submanifold W of A from which there is an embedding $r:W \to b(V)$ (where b(V) is the submanifold of T constructed in Lemma 3) such that $d(\bar{s}) \in W$, $W \subseteq d(S)$, $d^{-1}(W) \subseteq V$, and

$$\forall s \in d^{-1}(W) \quad u(s,r(d(s))) = \max_{a \in W} u(s,r(a)). \tag{7}$$

<u>Proof</u>: Since $\bar{s} \in I(d)$, there is a neighborhood of S contained in V whose image under d is a submanifold W of A and from which d is a diffeomorphism to W. For simplicity, it will be assumed that $V = d^{-1}(W)$. That V and W are diffeomorphic under d follows from Lemma 2 and [2, Prop.I.2.10]. Then (7) holds if $r:W \to b(V)$ is defined by $r = bd^{-1}$, by (6), and r is a diffeomorphism since it is a composition of diffeomorphisms. Q.E.D.

<u>Lemma 5</u>: If $k = m \leq n$ or if $k < m$ and $k < n$, and also $\bar{s} \in I(d)$ and u satisfies (1), (2) and (3), then there exist a neighborhood Z of $d(\bar{s})$ in W and a neighborhood Y of $d(\bar{s})$ in A and a smooth mapping $r:Y \to T$ such that $Z \subseteq Y$ and

$$\forall s \in d^{-1}(Z) \quad u(s,r(d(s))) = \max_{a \in Y} u(s,r(a)). \tag{8}$$

and such that d(s) is the unique maximizing action if $s \in Z \cap d^{-1}(Y)$.

<u>Proof</u>: If $k = m \leq n$, then W can be taken to be both the neighborhood Z and the neighborhood Y, and (7) is equivalent to (8).

Now suppose that $k < m$ and $k < n$. It is sufficient to extend the function r of Lemma 4 from a neighborhood Z of $d(\bar{s})$ in W to a neighborhood Y of $d(\bar{s})$ in A with $Z \subseteq Y$, taking care so that $a \notin Z$ implies $r(a) \notin r(Z)$. This feature of the extension, along with the strict concavity of u in t, will assure uniqueness on $d^{-1}(Z)$ of the maximizing action.

Since W is a k-dimensional submanifold of A, there exist a neighborhood E of $d(\bar{s})$ in A and an immersion $e:E \to \mathbb{R}^m$ which maps E diffeomorphically to its range and such that $e(d(\bar{s})) = \underline{0}$ and $W \cap E = \{e^{-1}(x) \mid \forall i > k \ x_i = 0\}$. Since b(V) is a k-dimensional submanifold of T, there exist a neighborhood F of $r(d(\bar{s}))$ in T and an immersion $f:F \to \mathbb{R}^n$ which maps F diffeomorphically to its range and such that $f(r(d(\bar{s}))) = \underline{0}$ and $b(V) \cap F = \{f^{-1}(y) \mid \forall j > k \ y_j = 0\}$. Define $C = E \cap r^{-1}f^{-1}(F)$. Note that C is a neighborhood of $d(\bar{s})$ in W.

Now define the projection $p:\mathbb{R}^m \to \mathbb{R}^m$ by $p(x) = (x_1,\ldots,x_k,0,\ldots,0)'$, and define $c:p^{-1}e(C) \to \mathbb{R}^n$ by $c(x) = (f^1re^{-1}p(x),\ldots,f^kre^{-1}p(x), \sum_{i=k+1}^{m} x_i^2,0,\ldots,0)'$. Finally, define $Y = e^{-1}c^{-1}(F)$ and $Z = W \cap Y$, and extend r from Z to Y by defining $r(a) = f^{-1}ce(a)$. Note that $r^{-1}r(Z) = Z$, which is sufficient for (8) to hold.

Q.E.D.

Proof of Theorem 1: Let u satisfy (1), (2) and (3), and let r be the neighborhood of $d(\bar{s})$ and the mapping from Y to T described in Lemma 5. Choose $\alpha > 0$ sufficiently small so that $|a - d(\bar{s})| < 2\alpha \Rightarrow a \in Y$, and define $Z = \{a| \; |a - d(\bar{s})| < \alpha\}$. A mapping $q:A \to T$ will be constructed such that $q(a) = r(a)$ for $a \in Z$, and such that $q(a) \notin r(Z)$ for $a \notin Z$. Thus q will strictly support d at $\bar{s}$ because, by (8), $d(s)$ is the unique action which maximizes $u(s,q(a))$ for $s \in d^{-1}(Z)$. The reward schedule q will be constructed by first constructing a smooth function $h:\mathbb{R}^m \to \mathbb{R}^m$ which is equal to the identity mapping on Z and which maps $\mathbb{R}^m \backslash Z$ to $Y \backslash Z$. Then q may be defined by $q(a) = rh(a)$.

Thus it is sufficient to construct h. Begin with a smooth mapping $g:\mathbb{R} \to [0,1]$ such that $g(\beta) = 1$ for $\beta \leq \alpha$ and $g(\beta) = 0$ for $\beta \geq 2\alpha$. Such a mapping exists by [2, Cor.I.4.7]. In terms of this mapping, h may be defined by

$$h(x) = \left(\int_0^{|x-d(\bar{s})|} g(\beta)d\beta/|x - d(\bar{s})|\right)(x - d(\bar{s})) + d(\bar{s}).$$

Note that $|h(x) - d(\bar{s})| < 2\alpha$ for all $x \in \mathbb{R}^m$, so that $h:\mathbb{R}^m \to Y$, and

that the coefficient of $(x - d(\bar{s}))$ in the definition is 1 if $x \in Z$ and is at least $\alpha/|x - d(s)|$ if $x \notin Z$. Q.E.D

Proof of Theorem 2: Suppose that u satisfies (4) and (5), and consider any transfer t. By (5), $u_T(\bar{s},t) \neq 0$. Thus there is a unique hyperplane X in $\mathbb{R}^n$ which contains t and to which $u_T(\bar{s},t)$ is a normal vector. Let $v:S \times X \to \mathbb{R}$ be the restriction of u to $S \times X$. Now Theorem 1 applies to v, with $X = T$ and $t = \bar{t}$, so there exists a reward schedule $r:A \to X$ which locally supports d for v at $\bar{s}$. Since u and v coincide at every pair $(s,r(a))$, r locally supports d for u at $\bar{s}$ as well.

It may be verified in a straightforward way that the subset of U on which (4) and (5) are satisfied is open in the Whitney C^2 topology. This set is nonempty, since it contains the function defined by $u(s,t) = -[\sum_{i=1}^{k} \exp(s_i + t_i) + \sum_{i=k+1}^{n} \exp(t_i)]$. Q.E.D.

Proof of Theorem 4: Suppose that $d(s) = \bar{a}$ for all states s in some neighborhood W of $\bar{s}$. Let X be the set of utility functions u such that, for all $t = T$, $u_T(\bar{s},t) \neq \underline{0}$. X is clearly nonempty, and it is routinely shown that X is open in the Whitney C^2 topology on U.

Suppose that $u \in X$, and choose a transfer $\bar{t}$. It is sufficient to construct a reward schedule q which strictly supports d for u at $\bar{s}$ and which satisfies $r(\bar{a}) = \bar{t}$. By continuity of u_T, there are neighborhoods Y of $\bar{s}$ and Z of $\bar{t}$ such that $(u_T(s,t))'(u_T(\bar{s},\bar{t})) > 0$ for $s \in Y$ and $t \in Z$. It may be assumed without loss of generality that

$Y \subset W$. Choose a positive constant α sufficiently small that a ball of radius $\alpha|u_T(\bar{s},\bar{t})|$ centered at $\bar{t}$ is contained in Z. Now define $h:A \to \mathbb{R}$ by

$$h(a) = ((a - \bar{a})'(a - \bar{a}))((a - \bar{a})')(a - \bar{a}) + 1)^{-1}$$

and define a reward schedule r by

$$r(a) = \bar{t} - \alpha h(a) u_T(\bar{s},\bar{t})$$

A path integration shows that r strictly supports d for u on Y.

Q.E.D.

5. PROOF OF THEOREM 3

The idea behind this proof is simple. If $\bar{s} \in L(d)\backslash I(d)$, then $\bar{s}$ lies on a manifold in S along which d is constant. If r strictly supports d for u at $\bar{s}$, then $d(\bar{s})$ must satisfy the first-order condition for maximization of u not only at $\bar{s}$, but everywhere on some interval of the path. This first-order condition is that $u_T(s,r(d(s)))'r_A(d(s)) = \bar{0}$. The first-order condition implies that $u_T(s,d(\bar{s}))$ is orthogonal to the columns of $r_A(d(\bar{s}))$ for every state s on the manifold where $d(s) = d(\bar{s})$. If $r_A(d(\bar{s}))$ could be assured to be nonzero, then the condition would constrain $u_T(s,d(\bar{s}))$ to map the constant manifold of d into a lower- dimensional submanifold of $\mathbb{R}^n$, and this constraint is obviously very restrictive. The technical problems in the proof are to show that the constraint is binding even if $r_A(d(\bar{s})) = \underline{0}$ and to establish a precise topological criterion for

how restrictive it really is.

Let d such that $\bar{s} \in L(d)\backslash I(d)$ be given. By Lemma 2, d maps a neighborhood of $\bar{s}$ surjectively to a submanifold of A. There are coordinate systems on neighborhoods of $\bar{s}$ and $d(\bar{s})$ which permit d to be treated as a linear mapping:

<u>Lemma 6</u>: Suppose that $\bar{s} \in L(d)$ and that rank $d_S(\bar{s}) = j$. Define $\pi:\mathbb{R}^k \to \mathbb{R}^m$ by $\pi(s) = (s_1,\ldots,s_j,0,\ldots,0)'$. Then there are coordinate systems on a neighborhood Y of $\bar{s}$ and on a neighborhood Z of $d(\bar{s})$ in A such that, when d is expressed in these coordinates, $d = \pi$ on Y.

<u>Proof</u>: This is a consequence of [2, Cor.I.2.6] and [2, Defn.I.2.7]. The application of [2, Cor.I.2.6] is justified by Lemma 2. Q.E.D.

From now on, Y and Z will be referred to by the coordinates described in Lemma 6. Furthermore it will be assumed that $\bar{s} = \underline{0} \in \mathbb{R}^k$ and that $Y = (-1,1)^k$. It should be clear that no generality will be lost by this. Define a mapping $y:(-1,1)^2 \to Y$ by $y(\alpha,\beta) = (\alpha,0,\ldots,0,\beta)'$. In particular, $y(0,0) = \bar{s}$. Define $w(\beta) = y(0,\beta)$. Also, for each positive integer h, define $I_h = [0,h^{-1}]$.

<u>Lemma 7</u>: If $\bar{s} \in L(d)$ and r strictly supports d for u at $\bar{s}$, then for every h there exists $\alpha_h \in I_h$ such that $r_1(d(y(\alpha_h,0))) \neq \underline{0}$.

<u>Proof</u>: If r strictly supports d for u at $\bar{s}$, then $u(\bar{s},r(d(y(h^{-1},0)))) < u(\bar{s},r(d(y(0,0))))$. This implies that

$r(d(y(h^{-1},0))) \neq r(d(y(0,0)))$. By the fundamental theorem of calculus, $r(d(y(h^{-1},0))) - r(d(y(0,0))) =$
$\int_0^{h^{-1}} r_A(d(y(\alpha,0)))d_S(y(\alpha,0))y_1(\alpha,0)d\alpha = \int_0^{h^{-1}} r_1(d(y(\alpha,0)))d\alpha$. (This last equality is because, by choice of coordinates, $d_S(y(\alpha,0))y_1(\alpha,0) = (1,0,\ldots,0)'$.) In order for the difference to be nonzero, $r_1(d(y(\alpha,0))) \neq \underline{0}$ for some $\alpha \in I_h$. Q.E.D.

yields (7). Q.E.D.

Lemma 8: If $\bar{s} \in L(d)\backslash I(d)$ and $t \in T(d,\bar{s},u)$, then there exist a positive integer j and a vector c on the unit sphere C of $\mathbb{R}^n$ such that, for $\beta \in I_j$,

$$u_T(w(\beta),t)'c = 0. \qquad (9)$$

Proof: Suppose that r strictly supports d for u at $\bar{s}$ and that $r(\bar{s}) = t$. Then j may be chosen so that r strictly supports d for u at every state in I_j^n. The first-order condition for $u(y(\alpha_h,\beta),r(a))$ to attain its maximum at $d(y(\alpha_h,\beta)) = a$ implies that

$$u_T(y(\alpha_h,\beta),r(d(y(\alpha_h,\beta))))r_1(d(y(\alpha_h,\beta))) = 0. \qquad (10)$$

Since $d(y(\alpha_h,\beta)) = d(y(\alpha_h,0))$, (10) is equivalent to

$$u_T(y(\alpha_h,\beta),r(d(y(\alpha_h,0))))'r_1(d(y(\alpha_h,0))) = 0. \qquad (11)$$

By assumption, then, (11) holds for $h \geq j$ and $\beta \in I_j$. By Lemma 7, $|r_1(d(y(\alpha_h,0)))|^{-1}r_1(d(y\alpha_h,0)))$ is a well defined element of C, and

because C is compact it may be assumed without loss of generality that these vectors converge to a unit vector c as h tends to infinity. Taking the limit of (11) as h increases yields (9). Q.E.D.

Lemma 9: If $t \in T(d,\bar{s},u)$, then there is a positive integer h such that the vectors $u_T(w(ih^{-1}),t)$ are linearly dependent for $1 \leq i \leq n$.

Proof: By Lemma 8, j and c can be selected so that (9) holds. Then, if $h = n^{-1}j$, the vectors $u_T(w(ih^{-1}),t)$ must all lie in the $(n-1)$-dimensional subspace of $\mathbb{R}^n$ orthogonal to c. Q.E.D.

Proof of Theorem 3: Let Z be a countable dense subset of T. For every positive integer h and z in Z, let X_{hz} be the set of utility functions u for which the vectors $u_T(w(ih^{-1}),z)$ are linearly independent for $1 \leq i \leq n$. Define X to be the intersection of all of the X_{hz} (of which there are only countably many). It is routinely established that each X_{hz} is a dense open set in the Whitney C^2 topology on U, so X is a residual set. Now it will be shown that, for each u in X, the complement of $T(d,\bar{s},u)$ contains a subset which is residual in T. This fact establishes the theorem.

Consider a utility function u which is in X. For every $h \geq 1$ and $z \in Z$, let Y_h be the set of transfers t such that the vectors $u_T(w(ih^{-1}),t)$ are linearly independent for $1 \leq i \leq n$. it is routinely established that Y_h is open in T. Furthermore, because $u \in X_{hz}$ for every z, $Z \subset Y_h$. Therefore, if Y is defined to be the intersection of the sets Y_h, then Y is a residual subset of T. If $t \in Y$, then the vectors $u_T(w(ih^{-1}),t)$ are linearly independent for $1 \leq i \leq n$, for every positive h. Therefore, by Lemma 9, $t \notin T(d,\bar{s},u)$. Q.E.D.

6. CONCLUSION

This paper has dealt in a preliminary fashion with a question about the practical significance of the revelation principle in the theory of incentives. This is the question of whether the reporting of private information to which the principle refers must literally be exhaustive or whether it is sufficient that summary information directly relevant (according to some reasonably strict standard of relevance) should be disclosed. The explicit results of the paper concern the decision problem of a single person facing an incentive schedule, rather than referring directly to the interrelated decision problems of several players in a game. Nonetheless, these results suggest that the distinction between complete reporting and summary reporting (or, more generally, between decision rules which are immersions and those which are not) should be taken very seriously.

The principal result of the paper is Theorem 3 which asserts that, precisely because of the geometrical feature which makes it informationally efficient, the decentralizability of summary reporting will be at best an extremely fragile property for almost all utility functions. Only under stringent restrictions like separability of utility will it likely be practical, even when the incentive-giver has considerable knowledge, to strictly support summary reporting. It might be thought that this result shows that strict supportability is an extraordinarily demanding incentive criterion, but Theorem 1 shows

that exhaustive reporting will meet this criterion in a reasonably broad set of circumstances. Thus, the two results together suggest that decentralization of summary reporting rules may be much harder to achieve than is exhaustive reporting. In this case, in the presence of informational costs and constraints, the revelation principle can no longer be regarded as a universal and exact solution to the problem of decentralization.

This paper has emphasized a negative result. Several avenues of research are now open which, if successful, would mitigate this result. One such avenue is the study of parametric restrictions on preferences which would insure the strict supportability (at least locally) of summary reporting rules. Certain ways of construing preferences over transfers as derived preferences (e.g., if transfers are random variables, the assumption of expected-utility maximization) impose separability, so these may also have this favorable effect. Such restrictions describe sets of utility functions which are nowhere dense in U, but in many situations the restrictions have economic justification.

A related avenue of research is the study of approximate supportability of decision rules. It is likely that, if summary reporting is strictly supportable for separable preferences, then reporting summary information about some state close to the true state is strictly supportable for a utility function which is (in some sense) nearly separable. Thus, even if restrictions on preferences cannot be assumed to hold exactly, they might be used to decentralize rules which were at least approximately optimal.

The suggestion just made implies that another avenue of research, neglected during the past few years, might profitably be reopened. If it is recognized that, in practice, incentive schemes d constructed via the revelation principle will be only approximately efficient, then the study of various other ways to construct schemes designed only to achieve approximate efficiency is also a defensible enterprise. Such a study has been begun in [3].

NOTES

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1. Technically, the revelation principle applies to various noncooperative equilibrium concepts which have the property that persons' equilibrium actions are measurable with respect to the information which they possess. Cf. Laffont and Maskin [7].

2. Besides this paper, there are three other recent papers which have studied the interaction between communication requirements and incentives. First, Stefan Reichelstein [4] shows that, for dominant-strategy or Bayesian Nash implementation of social choice rules, the dimension of players' strategy spaces must be larger than it would have to be if straightforward behavior could be assumed. Second, simultaneously with the present work and independently of it, Jerry R. Green and Jean-Jacques Laffont [4] have derived analogues of the results presented here. Their work is done in the context of a class of quasilinear utility functions. Green and Laffont also investigate the question of how large a space of rewards is needed to strictly support a decision rule, and show that in general the reward space must have at least as high a dimension as the action space has. Third, James Gerard [1] has studied the issue of summary reporting in financial markets, using a generalization of the model due to Steven Ross

[10] as the basis for his work. In particular, the model concerns a firm manager who is assumed directly to be a risk-neutral income maximizer, rather than characterizing a decision maker's preferences indirectly by specifying preferences among reward schedules as in the present paper. Gerard shows that, within the parametric class of reward schedules considered by Ross, it is impossible to support strictly any one-dimensional decision rule which reveals the market value of a firm with a distribution of returns drawn from a two-parameter family.

3. It has just been pointed out that important issues are obscured in Ross's work by the assumption that decision-makers' private information is of a particularly simple form (i.e., that it is one dimensional). This observation is not meant to be a criticism specific to Ross. The simplifying assumption that states are somehow naturally ordered along a one-dimensional continuum is ubiquitous in the incentives literature. It has been used widely in the study of insurance contracts, auctions, predation and limit pricing, and a variety of other situations. In each of these applications, it involves what may be a considerable sacrifice of realism. For instance, differences in risk aversion may be of comparable importance to the differences in reservation price by which bidders' utilities are characterized in the usual model of auctions. In order to take account of the possibility that these characteristics of bidders may vary independently, a two-dimensional state space is required.

4. Of course, the comparison that is actually relevant is between the firm's annual report to shareholders and its 10K report. The complexity of reporting has been drastically understated in this discussion, in the interest of simplicity.

5. Several authors have established that, if information imperfectly correlated with a manager's conduct but provided independently of the manager is available without cost, then it is efficient to use it in determining his compensation. (Cf. B. Holmstrom [6] and the references given there.) That principle concerns the use of information in the statistical sense. I.e., the conditional distribution of the information, given the manager's conduct, is an exogenous feature of the environment. In contrast, the principle enunciated here concerns the use of a report about the content of which the manager will have complete discretion. In terms of a standard distinction in game theory (cf. [5]), the former principle concerns questions of imperfect information while the present one concerns incomplete information. Thus, while they bear a strong superficial resemblance because they both consider changes in the basis on which rewards or penalties are assigned, the two principles are not closely related.

6. The Whitney C^2 topology is formally defined and discussed in [2, Sec. II.3]. Intuitively, this is the coarsest topology in which every set of functions definable (in a continuous way) by pointwise strict inequalities involving the graph of a function

and its first and second derivatives. (E.g., the set of utility functions having nonsingular Hessian matrix at every point is open, since this condition is equivalent to the condition that the square of the determinant of the Hessian should be strictly positive at every point.) The sets asserted in this paper to be Whitney C^2-open will evidently satisfy this intuitive criterion, so formal proofs of openness will be omitted. In any topological space, a set which is the intersection of countably many dense open sets is called residual. A space has the Baire property if every residual set is dense. Both $\mathbb{R}^n$ under its usual topology and also U under the Whitney C^2 topology possess the Baire property.

7. A function is a submersion at a point if the rank of its Jacobian matrix there is equal to the dimension of its range. Thus, if A is the regarded as the range of d, $s \in L(d)$ need not imply that d is a submersion at s (unless $m = 1$). However, Lemma 2 below will assert that the restriction of d to some neighborhood of s is a submersion to a submanifold of A if $s \in L(d)$.

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